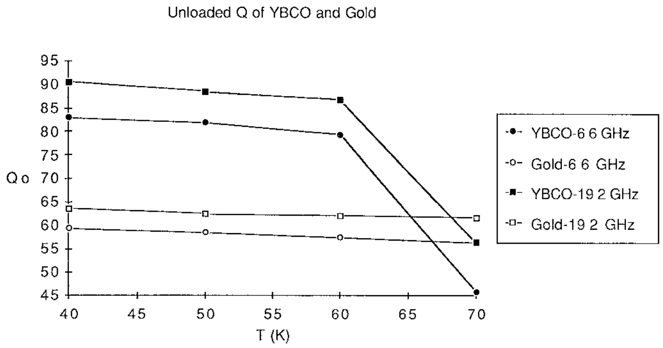

Fig. 2. Plot of unloaded Q versus temperature shows the high frequency transition below T_c for the YBCO on MgO resonator. The superconductor is better than gold below 50 K by about 40 percent.

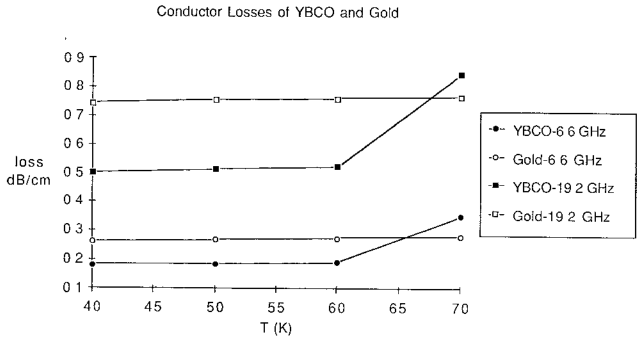

Fig. 3. Plot of conductor attenuation versus temperature shows that YBCO has lower loss than gold by about 32 percent on the MgO substrate.

sample was fabricated with a gold ground plane, a higher Q is expected for a superconducting ground plane.

Fig. 3 plots the temperature dependence of conductor attenuation α_c for the gold and YBCO samples. The conductor loss for YBCO is lower than gold by 31 percent at 6.6 GHz and 33 percent at 19.2 GHz at 40 K. The sharp rise in Q below T_c is qualitatively consistent with the two-fluid model. The relatively high attenuation and surface resistivity in both the gold and superconductor can be attributed to the fact that the film thickness is comparable to skin depth of the materials. Other factors could be the intergranular coupling and the anisotropic nature of the ceramic superconductors, which do not provide a good conducting path for high frequencies. The BSCCO sample was measured to have a slightly higher conductor loss than gold. The YBCO films grown on ZrO_2 substrates which were fabricated by another company did not yield films with low enough surface resistivity. The higher losses of these samples could be attributed to the degree of c-axis orientation of the films [9].

Preliminary studies on resonators with superconductors on both sides of the dielectric have been performed. Presently, the unloaded Q and conductor losses are close to the superconducting sample with the gold ground plane. Development of double-sided fabrication of HTS samples will further enhance the usefulness of HTS thin films to microstrip applications.

The results of this study have shown that the microwave performance of highly oriented films of HTS is better than conventional gold conductors. Also, the use of microstrips to measure the microwave losses demonstrates the capability of HTS films to be directly implemented into integrated circuits. Understanding how the processing of HTS thin films can be developed to promote low microwave attenuation is essential to the success of superconducting microstrip circuits [10].

Acknowledgment

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Determination of the Cutoff of the First Higher Order Mode in a Coaxial Line by the Transverse Resonance Technique

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***Abstract*—This paper uses the transverse resonance technique and static field considerations to determine cutoff for the first higher order mode in a coaxial line. The result, while approximate, agrees very well with the rigorous field-theory solution for all practical coaxial lines. The solution also has an immediate connection with the physics of the problem which makes the result obtained almost obvious.**

I. Theoretical Development

In [1] the transverse resonance technique is developed to permit determination of cutoff in waveguides having doubly connected cross sections. The method relies on inserting a reference

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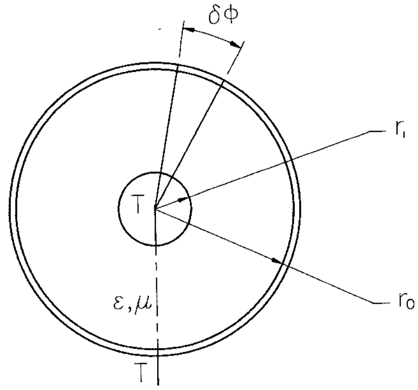

Fig. 1. Coaxial line cross section. At resonance, admittance at plane TT is zero.

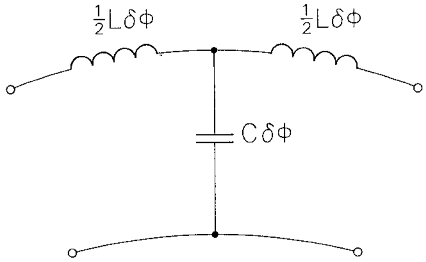

Fig. 2. Equivalent circuit.

plane at a convenient location within the cross section and then considering it as a two-port network folded back on itself. When the two-port thus exposed is itself a cascade of n identical and symmetrical elementary two-ports, each having a transmission matrix (a, b, c, d), then it is shown in [1] that the condition for resonance is

$$\cos\{n\cos^{-1}a\}=1. \tag{1}$$

To apply this to a coaxial line we choose any radius as the reference plane, as suggested in Fig. 1, and then divide the cross section into n segments of width $\delta\varphi$, where $n=2\pi/\delta\varphi\to\infty$. As in the circuit theory development of transmission line theory, each of these sectors may be represented by the equivalent network shown in Fig. 2. For this it is shown in any one of a number of texts (e.g. [2]) that

$$a=1-(1/2)LC(\omega\delta\varphi)^2 \tag{2}$$

where $L(C)$ is the inductance (capacitance) per radian per unit axial length.

Inserting this result into (1) and making approximations for small arguments gives as the condition for transverse resonance

$$\omega_c\sqrt{LC}=1. \tag{3}$$

Interpreted physically, (3) states that resonance will occur when the traveling wave in the cross section undergoes a phase shift of 2π per circuit.

To recast (3) in geometrical terms requires determination of L and C. This can be done from static considerations. For a wave circulating tangentially in the cross section, the electric field will be radially directed and the magnetic field axial.

Capacitance is obtained from the electrostatic problem of a pair of coaxial conducting cylinders, i.e.,

$$C=\epsilon/\ln(r_o/r_i) \tag{4}$$

where ϵ is the permittivity of the dielectric, and $r_o(r_i)$ is the outer (inner) radius of the coaxial line.

It is advantageous to use as alternative geometric parameters the mean radius $r_m=(r_o+r_i)/2$ and the interconductor gap width $t=(r_o-r_i)$. The Taylor expansion for logarithm then allows (4) to be restructured as

$$C=\epsilon r_m/St \tag{5}$$

where

$$S=1+(1/3)(t/2r_m)^2+(1/5)(t/2r_m)^4+\cdots. \tag{6}$$

To find the inductance a magnetostatic problem must be solved. For a wave circulating in the cross section, there will be equal and opposite sheet currents flowing on the inner and outer conductors. We consider the case where these are time invariant. Then it follows from Ampere's law that in the space between the conductors the magnetic field will be axial and uniform and the inductance per radian is easily shown to be

$$L=(1/2)\mu(r_o^2-r_i^2)=\mu r_m t \tag{7}$$

where μ is the permeability of the dielectric material.

Substituting (5) and (7) into (3) and noting that $\omega_c\sqrt{\mu\epsilon}=2\pi/\lambda_c$, λ_c being the cutoff wavelength, leads directly to the result

$$\lambda_c=2\pi r_m\{1-(1/6)(t/2r_m)^2-(7/120)(t/2r_m)^4-\cdots\}. \tag{8}$$

Equation (8) expresses in a very obvious way the well-known result that the cutoff wavelength for the H_{11} mode in a coaxial line is slightly smaller than the mean circumference.

II. Comparison with the Exact Solution

To do the problem in a rigorous field-theory way leads to a Bessel function transcendental equation from inspection of which nothing is immediately evident. This equation is given in Marcuvitz [3], where there is also a tabulation essentially of $\lambda_c/2\pi r_m$ against r_o/r_i. (These are the data contained in the second column of table 2.4 which relate to the H_{11} mode, and they must be divided by 2 and inverted to obtain exactly this result.) It is interesting to compare these results with those obtained from the solution given here. Agreement is better than 1 percent for $r_o/r_i<2.5$ and better than 4 percent for $r_o/r_i<4$, which more than covers all practical cases.

III. Conclusion

The method given here, while approximate, has the simplicity of requiring no transcendental equation to be solved and of having an immediate connection with the physics of the problem, making it almost obvious without even solving the problem at all that the cutoff wavelength can be expected to approximate the mean circumference.

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